MR2424827 05A18 (05E15 20F55)
Reading, Nathan (1-NCS)
Chains in the noncrossing partition lattice. (English summary)
SIAM J. Discrete Math. **22** (2008), *no. 3,* 875–886.

{A review for this item is in process.}

MR2383226 20F55 (05E15 55U10)
Tzanaki, Eleni (GR-CRET)
Faces of generalized cluster complexes and noncrossing partitions. (English summary)
SIAM J. Discrete Math. **22** (2008), *no. 1,* 15–30.

{A review for this item is in process.}

MR2366971 (2008k:20088) 20F55 (05E15)
Brady, Thomas (IRL-DCTY); **Watt, Colum** (IRL-DIT-SM)
Non-crossing partition lattices in finite real reflection groups. (English summary)
Trans. Amer. Math. Soc. **360** (2008), *no. 4,* 1983–2005.

Given a finite irreducible real reflection group W having reflection set $\mathcal{R}$, it is possible to define a partial order given by the reflection length function $\tilde{l}$: for $u, v \in W$, $u \leq v$ if and only if $\tilde{l}(v) = \tilde{l}(u) + \tilde{l}(u^{-1}v)$, where $\tilde{l}(w)$ is the smallest positive integer k such that w can be written as a product of k reflections from $\mathcal{R}$ (therefore $\tilde{l}$ is *not* the usual length function l).

For a finite real reflection group W the authors give a case-free proof, i.e., one that is independent of the classification of finite real reflection groups, that the closed interval $[I, \gamma]$, where I is the identity, and γ is a Coxeter element of W, viz. the product of all simple reflections of W (taken in any order, because different orders produce conjugate elements, so there is only one Coxeter element up to conjugacy), forms a lattice in the partial order on W induced by reflection length.

For this purpose the authors cleverly introduce a new simplicial complex which is a geometrical model for the partially ordered set $[I,\gamma]$. Furthermore, they prove that if n is the rank of W, this simplicial complex lies in the $(n-1)$-dimensional sphere, S^{n-1}, in $\mathbb{R}^n$, and its vertex set consists of a set of positive roots for W.

The paper is very well written and organized, and very pleasant to read.

Reviewed by *Alessandro Conflitti*

MR2336311 20F55 (05A18 05E15)

Reading, Nathan (1-MI)

Clusters, Coxeter-sortable elements and noncrossing partitions. (English summary)

Trans. Amer. Math. Soc. **359** (2007), *no. 12,* 5931–5958 (*electronic*).

{A review for this item is in process.}

MR2305586 (2008c:05005) 05A15 (05A18 05A19 05E15 20F55 52C07)

Athanasiadis, Christos A. (GR-CRET)

On some enumerative aspects of generalized associahedra. (English summary)

European J. Combin. **28** (2007), *no. 4,* 1208–1215.

Let Φ be a finite root system with W being its corresponding finite reflection group. Let L_W be the lattice of noncrossing partitions associated to W, and let $\Delta(\Phi)$ denote the cluster complex of Φ introduced in [J. Amer. Math. Soc. **15** (2002), no. 2, 497–529 (electronic); MR1887642 (2003f:16050)] by S. Fomin and A. Zelevinsky. In [Sém. Lothar. Combin. **51** (2004/05), Art. B51b, 16 pp. (electronic); MR2080386 (2005e:17013)], F. Chapoton introduced generating functions $F(\Phi)$ and $M(W)$ and conjectured that

$$(1-y)^n\, F\left(\frac{x+y}{1-y},\frac{y}{1-y}\right) = M\left(-x,\frac{-y}{x}\right).$$

This conjecture encapsulates many known similarities between the enumerative properties of

$\Delta(\Phi)$ and L_W.

The paper under review proves this conjecture in a case-free manner using ideas based on the work of T. Brady and C. Watt in ["Non-crossing partition lattices in finite real reflection groups", Trans. Amer. Math. Soc., to appear].

Reviewed by *Satyan L. Devadoss*

MR2272140 (2007i:05015) 05A18 (05A15 05E10)

Chen, William Y. C. (PRC-NNK-PMC); **Deng, Eva Y. P.** (PRC-NNK-PMC);
Du, Rosena R. X. (PRC-NNK-PMC); **Stanley, Richard P.** (1-MIT);
Yan, Catherine H. (1-TXAM)

Crossings and nestings of matchings and partitions. (English summary)

Trans. Amer. Math. Soc. **359** (2007), *no. 4,* 1555–1575 (*electronic*).

Let $[n]$ denote the set $\{1, 2, \ldots, n\}$. The standard representation of a partition P of $[n]$ is the graph on the vertex set $[n]$ whose edge set consists of arcs connecting the elements of each block of P in numerical order. For $k \geq 2$, a k-crossing (respectively, a k-nesting) of P is a k-element subset $(i_1, j_1), \ldots, (i_k, j_k)$ of the arcs in the standard representation of P such that $i_1 < \cdots < i_k < j_1 < \cdots < j_k$ (respectively, $i_1 < \cdots < i_k < j_k < \cdots < j_1$). Let $\mathrm{cr}(P)$ (respectively, $\mathrm{ne}(P)$) be the maximal j such that P has a j-crossing (respectively, a j-nesting). Let $\min(P)$ (respectively, $\max(P)$) be the set of minimal (respectively, maximal) block elements of P. Finally, for subsets S and T of $[n]$, let $f_{n,S,T}(i, j)$ be the number of partitions of $[n]$ such that $\mathrm{cr}(P) = i$, $\mathrm{ne}(P) = j$, $\min(P) = S$, and $\max(P) = T$. Then the authors' main result is that $f_{n,S,T}(i, j) = f_{n,S,T}(j, i)$. That is, the statistics $\mathrm{cr}(P)$ and $\mathrm{ne}(P)$ have a symmetric joint distribution. Many corollaries follow immediately, e.g., the number of k-noncrossing partitions of $[n]$ equals the number of k-nonnesting partitions of $[n]$. Earlier work by various other authors on the cases $k = 2$ and $k = 3$ is subsumed and vastly generalized.

The proof uses a bijection between set partitions and vacillating tableaux. A vacillating tableau V_λ^{2n} of shape λ and length $2n$ is a sequence $\lambda^0, \lambda^1, \ldots, \lambda^{2n}$ of integer partitions such that (i) $\lambda^0 = \varnothing$ and $\lambda^{2n} = \lambda$, (ii) λ^{2i+1} is obtained from λ^{2i} by doing nothing or deleting a square, and (iii) λ^{2i} is obtained from λ^{2i-1} by doing nothing or adding a square. Vacillating tableaux are related to the irreducible representations of the so-called partition algebra just as standard Young tableaux are related to the irreducible representations of the symmetric group. The key to the authors' proof is a bijection between the set of vacillating tableaux of empty shape and length $2n$ and the set of partitions of $[n]$. Under this bijection, $\mathrm{cr}(P)$ becomes the greatest number of rows in any λ^i, and $\mathrm{ne}(P)$ becomes the greatest number of columns in any λ^i. Thus the "obvious" duality between rows and columns yields the less obvious duality between crossings and nestings.

The special case $n = 2m$ and $|S| = |T| = m$ yields a duality between crossings and nestings for complete matchings. In this case, the vacillating tableaux specialize to oscillating tableaux, which are related to irreducible representations of the Brauer algebra.

The authors also consider a variant of their construction, which proves a duality between enhanced k-crossings and k-nestings, using a bijection with hesitating tableaux.

The paper concludes with an application to the problem of enumerating k-noncrossing (or k-nonnesting) matchings.

Reviewed by *Timothy Y. Chow*

MR2262893 (2007j:05221) 05E25 (05A18 06A07)

Athanasiadis, Christos A. (GR-CRET); **Brady, Thomas** (IRL-DCTY);
Watt, Colum (IRL-DIT-SM)

Shellability of noncrossing partition lattices. (English summary)

Proc. Amer. Math. Soc. **135** (2007), *no. 4,* 939–949 (*electronic*).

Associated to any finite Coxeter group W is a noncrossing partition lattice, denoted NC_W, which is defined as follows. Partially order elements of W by setting $u \le v$ whenever there exists a shortest factorization of u as a product of reflections in W which is a prefix of such a shortest factorization for v. Then NC_W is the interval in this partially ordered set (poset) from 1 to any Coxeter element γ, which is well-defined because all such intervals are isomorphic.

This paper gives a case-free proof that NC_W is EL-shellable. Specifically, covering relations are labeled in a natural manner by reflections, and these are ordered using a reflection ordering, namely a total order on reflections such that $t_{\alpha_1} < t_\alpha < t_{\alpha_2}$ or $t_{\alpha_2} < t_\alpha < t_{\alpha_1}$ whenever $\alpha, \alpha_1, \alpha_2$ are distinct roots with α a positive combination of α_1, α_2.

NC_W was previously proven EL-shellable in type A by A. Björner and P. Edelman [see A. Björner, Trans. Amer. Math. Soc. **260** (1980), no. 1, 159–183; MR0570784 (81i:06001)], and in type B by V. Reiner [Discrete Math. **177** (1997), no. 1-3, 195–222; MR1483446 (99f:06005)], while other cases were open prior to this case-free proof in the paper now under review.

Reviewed by *Patricia L. Hersh*

MR2263728 (2008d:05014) 05A18 (05E25 06A07 52B70)
Thomas, Hugh (3-FLDS2)
Tamari lattices and noncrossing partitions in type B. (English summary)
Discrete Math. **306** (2006), *no. 21,* 2711–2723.

Summary: "The usual, or type A_n, Tamari lattice is a partial order on T_n^A, the triangulations of an $(n+3)$-gon. We define a partial order on T_n^B, the set of centrally symmetric triangulations of a $(2n+2)$-gon. We show that it is a lattice, and that it shares certain other nice properties of the A_n Tamari lattice, and therefore that it deserves to be considered the B_n Tamari lattice. We also define a bijection between T_n^B and the noncrossing partitions of type B_n defined by V. Reiner [Discrete Math. **177** (1997), no. 1-3, 195–222; MR1483446 (99f:06005)]."

MR2252931 (2007c:05015) 05A18 (57M07 60C05)
McCammond, Jon (1-UCSB)
Noncrossing partitions in surprising locations.
Amer. Math. Monthly **113** (2006), *no. 7,* 598–610.

From the introduction: "Certain mathematical structures make a habit of turning up in the most diverse of settings. Some obvious examples exhibiting this intrusive type of behavior include the Fibonacci numbers, the Catalan numbers, the quaternions, and the modular group. In this article, the focus is on a lesser known example: the noncrossing partition lattice. The focus of the article is a gentle introduction to the lattice itself in three of its many guises: as a way to encode parking functions, as a key part of the foundations of noncommutative probability, and as a building block for a contractible space acted on by a braid group. Since this article is aimed primarily at nonspecialists, each area is briefly introduced along the way."

MR2218819 (2007a:20035) 20F55 (05A18 05E15 06A07 20F05 20F36)
Bessis, David (F-ENS-DAM); **Corran, Ruth** (F-PARIS6-I)
Non-crossing partitions of type (e,e,r). (English summary)
Adv. Math. **202** (2006), *no. 1,* 1–49.

The authors construct a Garside structure for the braid group $B(e,e,r)$ by generalising the notion of non-crossing partitions. Hence, their Garside structure can be seen as a generalisation of the dual Garside structures for the types A_n, D_n and $I_2(e)$.

A lattice $\mathrm{NCP}(e,e,r)$ of non-crossing partitions of type (e,e,r) as well as a complement operation and a height function on this lattice are defined in a purely combinatorial way, although the constructions are motivated by geometric properties of the complex reflection group $G(e,e,r)$. In brief, a non-crossing partition $p\in\mathrm{NCP}_V$ of a set $V\in\mathbb{C}$ of points is defined as a partition of V for which the convex hulls of its parts are pairwise disjoint; the set NCP_V is a lattice with respect to refinement. Then, $\mathrm{NCP}(1,1,e)$ is defined as $\mathrm{NCP}_{\{\mu_e\}}$, where $\{\mu_e\}$ is the set of e-th roots of unity, $\mathrm{NCP}(e,1,n)$ is the set of elements of $\mathrm{NCP}(1,1,en)$ which are fixed under multiplication by e-th roots of unity, and $\mathrm{NCP}(e,e,n+1)$ is the set of non-crossing partitions of $\mu_{en}\cup\{0\}$ for which "forgetting" the point 0 yields an element of $\mathrm{NCP}(e,1,n)$.

The authors then define natural maps $\alpha\colon \mathrm{NCP}(e,e,n+1)\to B(e,e,n+1)$ and $\beta\colon \mathrm{NCP}(e,e,n+1)\to G(e,e,n+1)$ which are shown to be poset isomorphisms, where $B(e,e,n+1)$ is endowed with the left-divisibility partial order and $G(e,e,n+1)$ with the prefix order with respect to decomposition of its elements into sequences of reflections.

Further, they prove that the monoid $M(e,e,n+1)$ generated by the image of α is a Garside monoid with group of fractions $B(e,e,n+1)$. That is, $B(e,e,n+1)$ is a Garside group whose simple elements correspond to the non-crossing partitions of type $(e,e,n+1)$.

An explicit presentation of $M(e,e,n+1)$, respectively $B(e,e,n+1)$, is given where the generators (the atoms in the Garside terminology) correspond to the non-crossing partitions of height 1 and the relations correspond to the non-crossing partitions of height 2. More precisely, the relations are $\alpha(u)\alpha(v)=\alpha(v)\alpha(v\backslash w)$, where $u,v,w\in\mathrm{NCP}(e,e,n+1)$, u and v are of height 1, w is of height 2, $u\preccurlyeq w$ and $v=u\backslash w$.

Reviewed by *Volker Gebhardt*

MR2220342 (2006m:05016) 05A18 (05A05 06A07 06D05)
Barcucci, Elena (I-FRNZ-I); **Bernini, Antonio** (I-FRNZ-I);
Ferrari, Luca [Ferrari, Luca[1]] (I-SIN-MI); **Poneti, Maddalena** (I-FRNZ-I)
A distributive lattice structure connecting Dyck paths, noncrossing partitions and 312-avoiding permutations. (English summary)
Order **22** (2005), *no. 4,* 311–328 (2006).

While there is an enormous amount of literature on Catalan-like objects, there is significantly less material on partially ordered sets of these. Though such partially ordered sets have been studied before, they were sometimes not even lattices, and they were almost never distributive (or even modular) lattices.

In this paper the authors define a natural partial ordering on the set of Dyck paths. If P and Q are two Dyck paths enumerated by C_n, then $P \leq Q$ if P stays weakly below Q. This partial order turns out to be a distributive lattice.

The authors show that this ordering can be translated into one on the set of noncrossing partitions of the set $[n]$. They also show that the poset is isomorphic to the poset of 312-avoiding permutations ordered by the strong Bruhat order.

Reviewed by *Miklós Bóna*

MR2087306 (2005d:06005) 06A07 (05D15)
Logan, Mark J. (1-MNM); **Shahriari, Shahriar** (1-PMN)
A new matching property for posets and existence of disjoint chains. (English summary)
J. Combin. Theory Ser. A **108** (2004), *no. 1,* 77–87.

This interesting paper is concerned with a new poset property, which, like many other poset properties, is a generalization of a property of the subset lattice.

Let P be a graded poset; let A and B be two subsets of P. We say that $f\colon A \to B$ is a matching of A into B if either for all $x \in A$, we have $x \leq f(x)$, or for all $x \in A$, we have $x \geq f(x)$.

Now let the shadow of x be the set of elements covered by x, and let the shade of x be the set of elements that cover x. Then we say that the graded poset P is shadow matching if, for all $x \in P$ and for all y from the shadow of x, there is an injective matching from the shadow of y to the shadow of x and there is an injective matching from the shade of x to the shade of y.

The authors prove that the divisor lattice, the subset lattice, the lattice of noncrossing partitions of $[n]$, and all geometric lattices are shadow matching. They prove that if a poset is shadow matching, and there is a bijective matching from the set A of elements of rank k into the set B of elements of rank l, then we can find $|A|$ disjoint saturated chains that start in elements of A and end in elements of B. This is called the Lehman-Ron property.

Reviewed by *Miklós Bóna*

MR1912816 (2003f:05129) 05E25 (05A17 06A07)
Biane, P. (F-ENS-DAM)
Parking functions of types A and B. (English summary)
Electron. J. Combin. **9** (2002), *no. 1, Note* 7, 5 *pp.* (*electronic*).

Summary: "The lattice of noncrossing partitions can be embedded into the Cayley graph of the symmetric group. This allows us to rederive connections between noncrossing partitions and parking functions. We use an analogous embedding for type B noncrossing partitions in order to answer a question raised by R. Stanley on the edge labeling of the type B non-crossing partitions lattice."

MR1766277 (2001g:05011) 05A15 (05A10 05A18 05A30 05E10 46L54 52B05)
Simion, Rodica (1-GWU)
Noncrossing partitions. (English, French summaries)
Formal power series and algebraic combinatorics (Vienna, 1997).
Discrete Math. **217** (2000), *no. 1-3,* 367–409.

A partition of $\{1, 2, \cdots, n\}$ is called noncrossing if, whenever there are four elements $1 \leq a < b < c < d \leq n$ such that a, c are in the same block and b, d are in the same block, then the two blocks coincide. The number of noncrossing partitions of an n-element set is the nth Catalan number $(n+1)^{-1}\binom{2n}{n}$. When G. Kreweras introduced noncrossing partitions in 1972, he certainly did not foresee how big the subject would grow, and how many fascinating connections to various other problems and fields would develop. Now, almost 30 years after their introduction, Simion provides us with a wonderful survey of the basic facts about noncrossing partitions and the lattice defined on them, and of their occurrences in- and outside of enumerative combinatorics. In particular, she touches upon their relevance in map enumeration and colouring, in the combinatorics of orthogonal polynomials, in the combinatorics of the associahedron and the permutahedron, their generalizations to other root systems, quasi-symmetric functions related to them, symmetric group actions defined on the space of maximal chains, their relevance in Voiculescu's theory of free probability, and their significance as an idealized model for the secondary RNA structure. It is a real pleasure to read this article. It contains several pointers to open problems and work that still needs to be done, and it ends with a long list of references.

{For the entire collection see MR1765535 (2001a:00029)}

Reviewed by *Christian Krattenthaler*

MR1644234 (99i:05204) 05E15 (05A18 20F55)

Athanasiadis, Christos A. (1-PA)

On noncrossing and nonnesting partitions for classical reflection groups. (English summary)

Electron. J. Combin. **5** (1998), *Research Paper* 42, 16 *pp.* (*electronic*).

Summary: "The number of noncrossing partitions of $\{1, 2, \cdots, n\}$ with fixed block sizes has a simple closed form, given by Kreweras, and coincides with the corresponding number for nonnesting partitions. We show that a similar statement is true for the analogues of such partitions for root systems B and C, defined recently by Reiner in the noncrossing case and Postnikov in the nonnesting case. Some of our tools come from the theory of hyperplane arrangements."

Reviewed by *Kimmo Eriksson* (Västerås)

MR1609945 (98k:05009) 05A18 (05C05)

Klazar, Martin (CZ-KARL-AM)

On trees and noncrossing partitions. (English summary)

Discrete Appl. Math. **82** (1998), *no. 1-3,* 263–269.

Summary: "We give a simple and natural proof of (an extension of) the identity $P(k, l, n) = P_2(k-1, l-1, n-1)$. The number $P(k, l, n)$ counts noncrossing partitions of $\{1, 2, \cdots, l\}$ into n parts such that no part contains two numbers x and y, $0 < y - x < k$. The lower index 2 indicates partitions with no part of size three or more. We use the identity to give quick proofs of the closed

formulae for $P(k,l,n)$ when k is 1, 2, or 3."

MR1468196 (98g:05016) 05A18
Liaw, S. C. (RC-NCT-AM); **Yeh, H. G.** (RC-NCT-AM); **Hwang, F. K.** (RC-NCT-AM); **Chang, G. J. [Chang, Gerard Jennhwa]** (RC-NCT-AM)

A simple and direct derivation for the number of noncrossing partitions. (English summary)

Proc. Amer. Math. Soc. **126** (1998), *no. 6,* 1579–1581.

Summary: "Kreweras considered the problem of counting noncrossing partitionings of the set $\{1,2,\cdots,n\}$, whose elements are arranged into a cycle in its natural order, into p parts of given sizes $n_1,n_2,\cdots,n_p$ (but not specifying which part gets which size). He gave a beautiful and surprising result with a proof which resorts to a recurrence relation. In this paper we give a direct, entirely bijective proof starting from the same initial ideal as Kreweras' proof."

MR1483446 (99f:06005) 06A07 (05A18 05E15)
Reiner, Victor (1-MN-SM)

Non-crossing partitions for classical reflection groups. (English summary)

Discrete Math. **177** (1997), *no. 1-3,* 195–222.

The lattice of set partitions of an n-set can be interpreted as the intersection lattice for the hyperplane arrangement in the root system of type A_{n-1}. Adopting this interpretation, we obtain natural analogues to the partition lattices for the classical reflection groups of types B and D. This paper introduces the type B and D analogues to the lattice of non-crossing partitions, $\mathrm{NC}^B(n)$ and $\mathrm{NC}^D(n)$, and studies the properties of these lattices. In so doing, the author collects many of the known results about the usual noncrossing partition lattice $\mathrm{NC}^A(n)$, summarizes the proofs of these results and generalizes them for types B and D. The poset $\mathrm{NC}^B(n)$ was first considered by Montenegro in a forthcoming paper, and the paper under review explains the equivalence of the two definitions. In studying the intervals of $\mathrm{NC}^D(n)$ a third family of posets emerges, $\mathrm{NC}^{BD}(n,S)$, for each $S\subseteq[n]$. These posets interpolate between the type B and D subposets in that $\mathrm{NC}^{BD}(n,\varnothing)=\mathrm{NC}^B(n)$ and $\mathrm{NC}^{BD}(n,[n])=\mathrm{NC}^D(n)$.

The author shows that the analogues to $\mathrm{NC}^A(n)$ are ranked lattices. Generalizing the work of R. E. Simion and D. H. Ullman [Discrete Math. **98** (1991), no. 3, 193–206; MR1144402 (92j:06003)], he shows that $\mathrm{NC}^B(n)$ is self-dual, and that $\mathrm{NC}^{BD}(n,S),\ \mathrm{NC}^B(n)$ and $\mathrm{NC}^D(n)$ have symmetric chain decompositions; he develops a recursion to count type B non-crossing partitions by various block statistics. Using parenthesization of certain infinite cyclic sequences, he generalizes work of P. H. Edelman [Discrete Math. **31** (1980), no. 2, 171–180; MR0583216 (81i:05018)] to give bijective proofs of formulas for the cardinalities, rank-generating functions, zeta polynomials, Möbius functions and rank-selected chain numbers for $\mathrm{NC}^{BD}(n,S)$, $\mathrm{NC}^B(n)$ and $\mathrm{NC}^D(n)$. By choosing an appropriate labelling of the atoms in the partition lattices of type BD, he provides EL-labellings for the partition lattices of type BD which restrict to EL-labellings of $\mathrm{NC}^{BD}(n,S)$ and computes the homotopy of intervals in the three lattices.

The enumerative formulas for type B non-crossing partitions are all remarkably similar to those for type A, and those for $\mathrm{NC}^{BD}(n,S)$ depend only on the cardinality of S. The author remarks on comparable results obtained for non-nesting partitions and suggests open problems related to the combinatorial and algebraic significance of the formulas obtained. He also proves the type B analogue to a not yet published result of Nica and Speicher, giving an incidence algebra approach to computing the zeta polynomial of $\mathrm{NC}^B(n)$.

Reviewed by *Darla Kremer*

MR1444167 (98m:05011) 05A17 (05E05)
Stanley, Richard P. (1-MIT)
Parking functions and noncrossing partitions. (English summary)
The Wilf Festschrift (Philadelphia, PA, 1996).
Electron. J. Combin. **4** (1997), *no. 2, Research Paper* 20, *approx*. 14 *pp*. (*electronic*).

Summary: “A parking function is a sequence $(a_1,\cdots,a_n)$ of positive integers such that if $b_1\le b_2\le\cdots\le b_n$ is the increasing rearrangement of $a_1,\cdots,a_n$, then $b_i\le i$. A noncrossing partition of the set $[n]=\{1,2,\cdots,n\}$ is a partition π of the set $[n]$ with the property that if $a<b<c<d$ and some block B of π contains both a and c, while some block B' of π contains both b and d, then $B=B'$. We establish some connections between parking functions and noncrossing partitions. A generating function for the flag f-vector of the lattice NC_{n+1} of noncrossing partitions of $[n+1]$ is shown to coincide (up to the involution ω on a symmetric function) with Haiman’s parking function symmetric function. We construct an edge labeling of NC_{n+1} whose chain labels are the set of all parking functions of length n. This leads to a local action of the symmetric group $\mathfrak{S}_n$ on NC_{n+1}.”

{For the entire collection see MR1444146 (98b:00038)}